Topics Covered

- Postulates
- Hermitian property of operators

Postulates

1. The state of a quantum mechanical system is completely specified by the wavefunction ψ**(r,t)** (in 3D) or ψ**(x,t)**.

   For the time independent cases which we will be mostly focusing on, we can drop the time dependence of the wavefunction.

   The wavefunction has to be normalised:

$$\int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx = 1$$

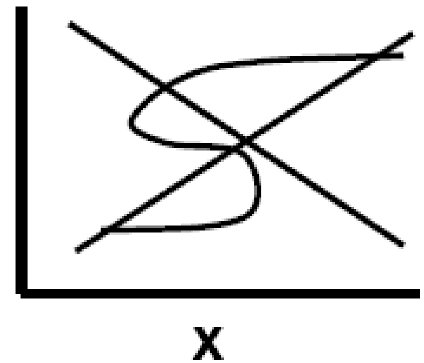
   If the wavefunction is not following the above relation, then introduce the **normalization constant 'N'**.

$$\int_{-\infty}^{\infty} N^*\psi^*(x)N\psi(x)dx = 1$$

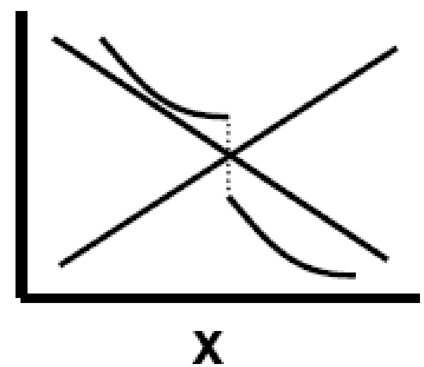
Required Properties of Ψ

Finite $\Psi \not\to \infty$ **(Square Integrable)**

Single Valued $\Psi(x)$

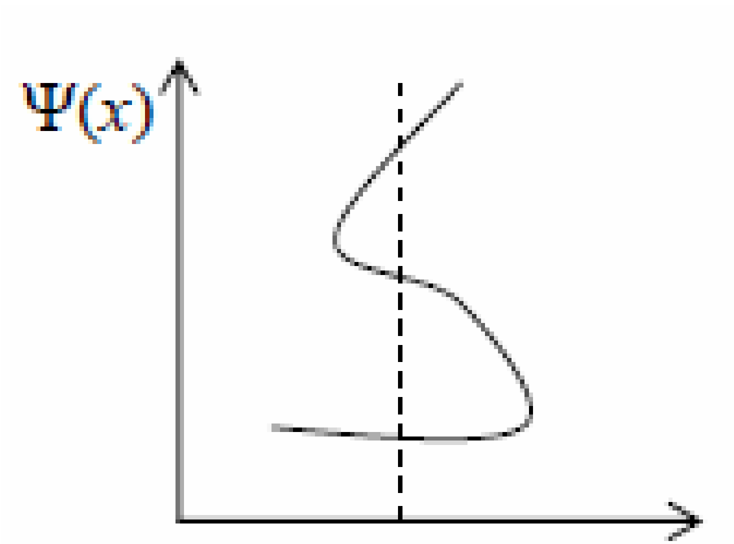


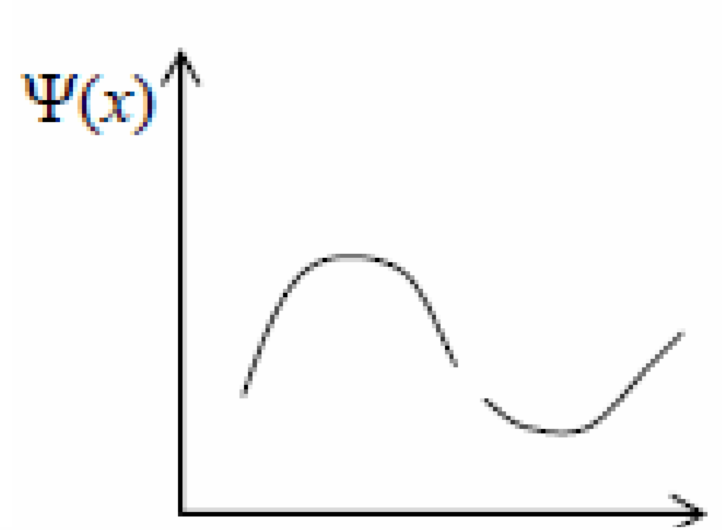
Continuous $\Psi(x)$

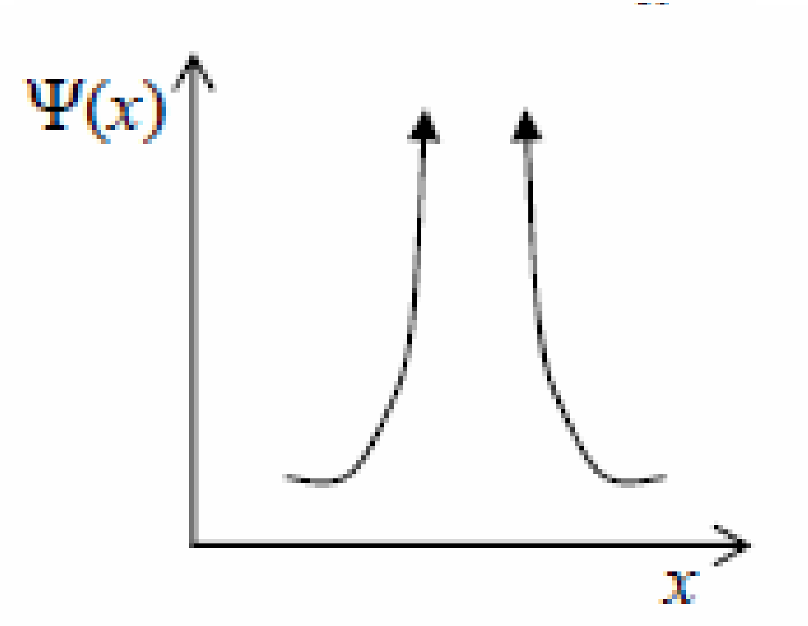


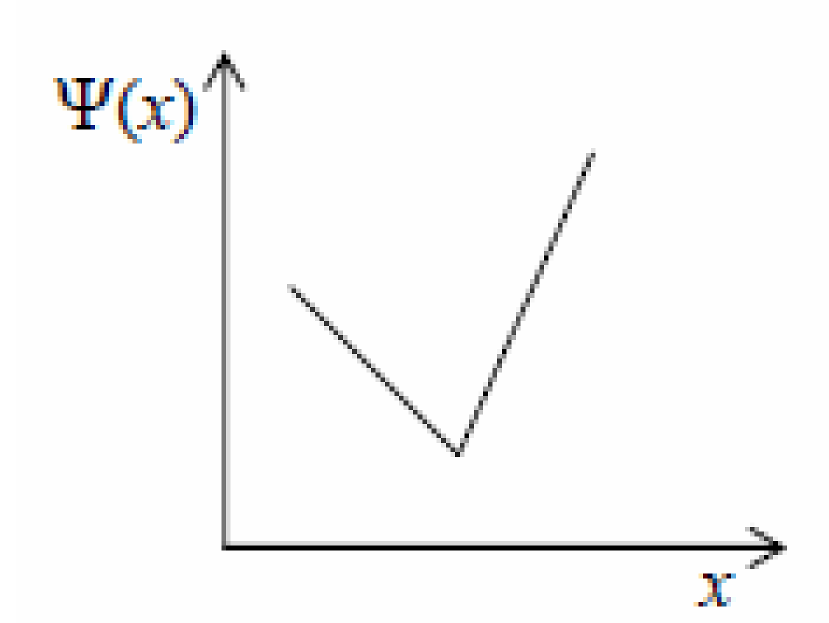
And derivatives must be continuous

Acceptable or Not ??

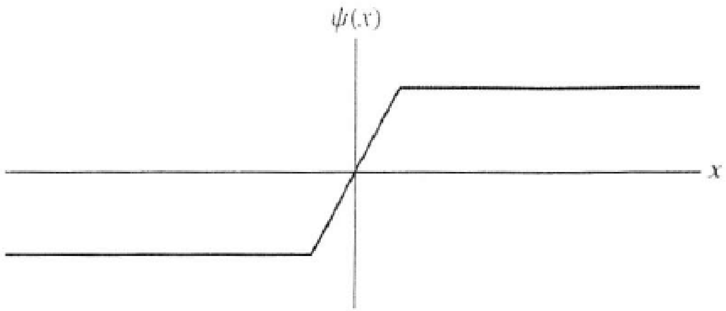




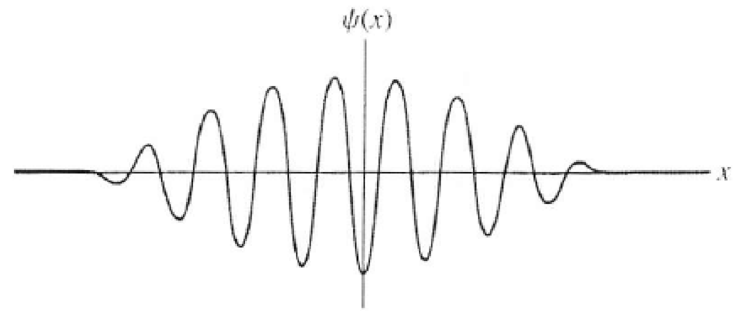




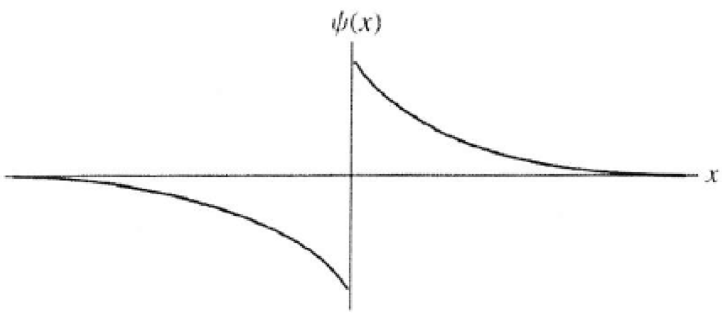
The graphs of several functions are shown below, For each, would this function be an acceptable wave function in quantum mechanics?



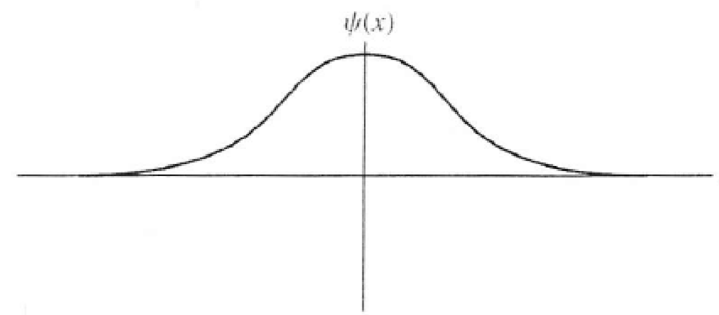
No. The wave function must approach zero as $x \to \infty$ and as $x \to -\infty$.



Yes.



No. The wave function must be continuous.



Yes.

Which of the following functions would be acceptable wavefunctions?

$e^{-x^2} \quad -\infty < x < +\infty$ **OK**

$e^{-x} \quad -\infty < x < +\infty$ **No - Diverges as x $\rightarrow$ -∞**

$\sin^{-1} x$ **No - Multivalued**
i.e. x = 1, $\sin^{-1}(1) = \pi/2, \pi/2 + 2\pi, \ldots$

$e^{-|x|} \quad -\infty < x < +\infty$ **No - Discontinuous first derivative at x = 0.**

2. To every observable (measurable property) in classical mechanics, there corresponds an operator in quantum mechanics.

All operators in quantum mechanics are **linear** and **Hermitian**

3. In any single measurement of the observable that corresponds to an operator Â, the only values that will ever be measured are the **eigenvalues** of that operator.

$$\hat{A}\psi_n = a_n \, \psi_n$$

4. If the system is in a state described the wavefunction $\psi(x)$, then the average or expectation value of the observable corresponding to the operator, Â, is given by

$$<a> = \frac{\int \psi^* \hat{A} \psi d\tau}{\int \psi^* \psi d\tau} = \int \psi^* \hat{A} \psi d\tau$$

If ψ is normalized

It is straightforward to show that If ψ_a is eigenfunction of Â with eigenvalue, a, then:

$<a> = a$

$<a^2> = a^2$

$\sigma_a = 0$ (i.e. there is no uncertainty in a)